Resonant Wire Antenna Efficiency

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Introduction

This concise paper attempts to summarise the most important results (for radio amateurs) of my recent investigations of resistive loss in wire antenna structures brought about by the confinement of the RF current flow to a thin region near the surface of the metal of the wires, and consequent dissipation of supplied power in the thin layer of metal that actually carries the current.

Scaling laws

Of interest is the way in which the properties of an antenna scale. For example, we know that for the overall size of a given design of Yagi the linear dimensions scale inversely with frequency. This means that if we double the frequency, the boom length halves, and the antenna element lengths halve. If we are using a standard NEC modelling package, we also halve the diameters of the tube that the elements are made from. (If we triple the frequency, the lengths all become 1/3, and so on).

Loss in an antenna scales in a more subtle way. To look into this problem, we need to investigate how the effective resistivity scales, and therefore how the cross sectional area of metal carrying the current scales. To do this, we need to investigate the skin depth.

Looking at the standard textbook formula for the skin depth δ which is (see for example, Wikipedia "skin depth")

$$\delta = \sqrt{2/(\omega\sigma\mu)}$$

where the symbols have the following meanings

angular frequency = $\omega = 2\pi$ f radians/sec
frequency = f Hz
electrical conductivity = σ Siemens per metre
electrical resistivity = $1/\sigma = \rho$ Ohms metres
magnetic permeability = $\mu = \mu_o\mu_r = (4\ \pi\ 10^{-7})\,\mu_r$ Henries per metre

The units of skin depth are metres.

We can rewrite this formula in a number of ways

$$\delta = \sqrt{(2\rho)/(\omega\mu)} = \sqrt{\rho/(\pi f\mu)} = \sqrt{(\rho\lambda)/(\pi c\mu)}$$

where we have used λf = c = (3x10^8) metres per second and λ is the wavelength in metres.

Also, because c$\mu_o = Z_o$ = 377 ohms = 120 π ohms we can rewrite this again as

$$\delta = (1/\pi)\sqrt{(\rho\lambda)/(120\mu_r)}$$

which isn't bad for a little algebraic manipulation. We now have the skin depth in terms of the wavelength (or frequency), the relative permeability μ_r of the material and the resistivity ρ of the material. To summarise in words, the skin depth scales as the square root of the wavelength, as the square root of the resistivity, and as the inverse square root of the relative permeability. For our purposes, the important point is the square root behaviour, for it means that the skin depth is not overly sensitive to small variations in these properties, and so it is not so important to have really precise and accurate estimates of them.

Just to check that we have done the algebra correctly, the internet sources agree on a skin depth in pure copper of about 9.3mm at a frequency of 50Hz where the wavelength is 6 million metres, the relative permeability is 1 and the electrical resistivity is 17.2 nano ohms metres. Doing the numbers,

$$\delta = (1/3.142)\sqrt{(17.2)(6)/(120{,}000)} = 9.33 \text{ mm}$$

so we now know how to do this kind of sum.

We can immediately make a table for the skin depths in pure copper at wavelengths of interest to radio amateurs.

Wavelength	Frequency	Skin depth
160m	1.875MHz	48.2 microns
80m	3.75MHz	34.1 microns
40m	7.5MHz	24.1 microns
20m	15MHz	17.0 microns
10m	30MHz	12.0 microns
2m	150MHz	5.4 microns
	Pure copper	

We recall that a micron is a millionth of a metre, or a thousandth of a millimetre, and that there are about 25.4 microns to the "thou" or "mil" if you need inches and depending which variant of English you speak.

The importance of the skin depth

In wires and tubes at HF frequencies, it is nearly always the case that the skin depth is smaller than the radius of the wire, or the thickness of the tube wall, as suggested by **Figure 1**. If this is the case, the current being confined to a layer one skin depth deep from the surface of the wire or tube, is carried by a conductor of effective cross sectional area $\pi D \delta$ where D is the diameter of the wire or tube and δ is the skin depth. This can be quite important as far as the resistive loss is concerned, for much of the metal in the wire carries no current and is just there for structural support. In that case, it can be made of some other material altogether, as we see in the use of copper plated steel or copper coated stainless steel antenna wire.

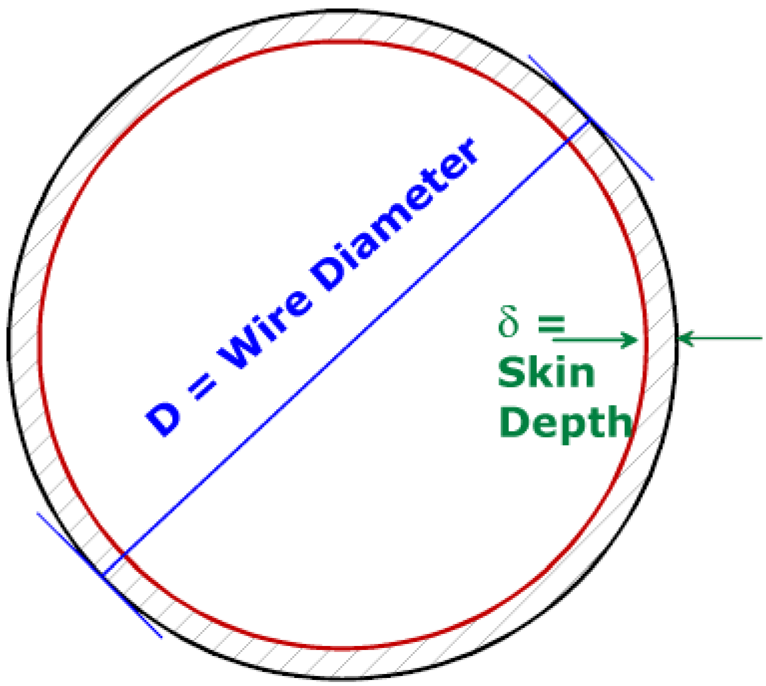

Figure 1
Sketch skin depth 12AWG wire

For wire of non-circular cross section, or woven or stranded wires, the cross sectional current-carrying area of interest is $P\delta$, where P is the perimeter (in metres) of the air-metal boundary at the outside of the bulk wire cross section. See **Figure 2**.

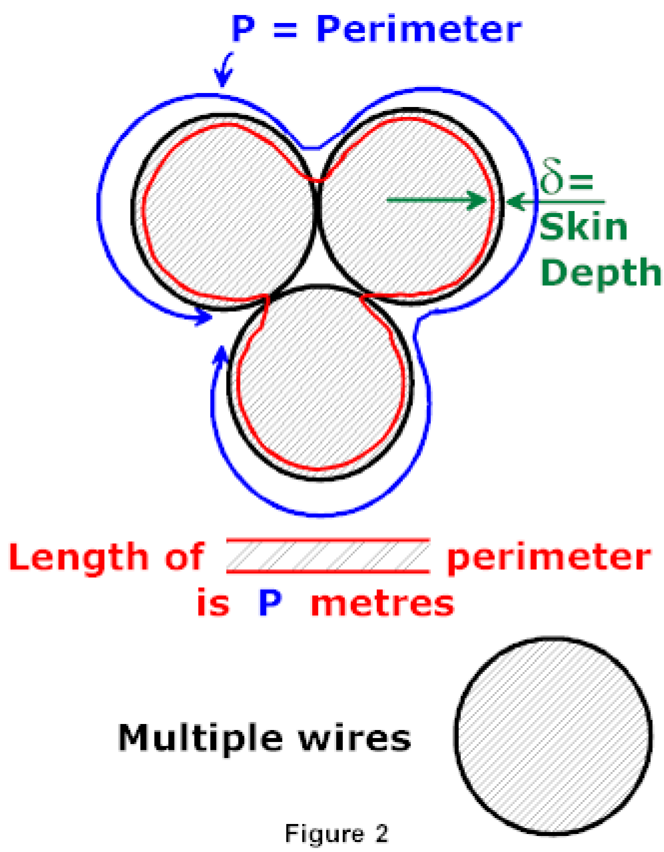

Figure 2
Sketch skin depth three 12AWG wires

Given uniform current along a length L of wire of diameter D and having skin depth δ, and knowing the resistivity ρ of the bulk metal, we can find the "effective resistance" of the wire by using the formula which relates resistance R ohms to resistivity ρ ohm-metres and cross section A square metres

$$R = \rho L / A = \rho L / (\pi D \delta)$$

Resistive loss in half-wave antennas

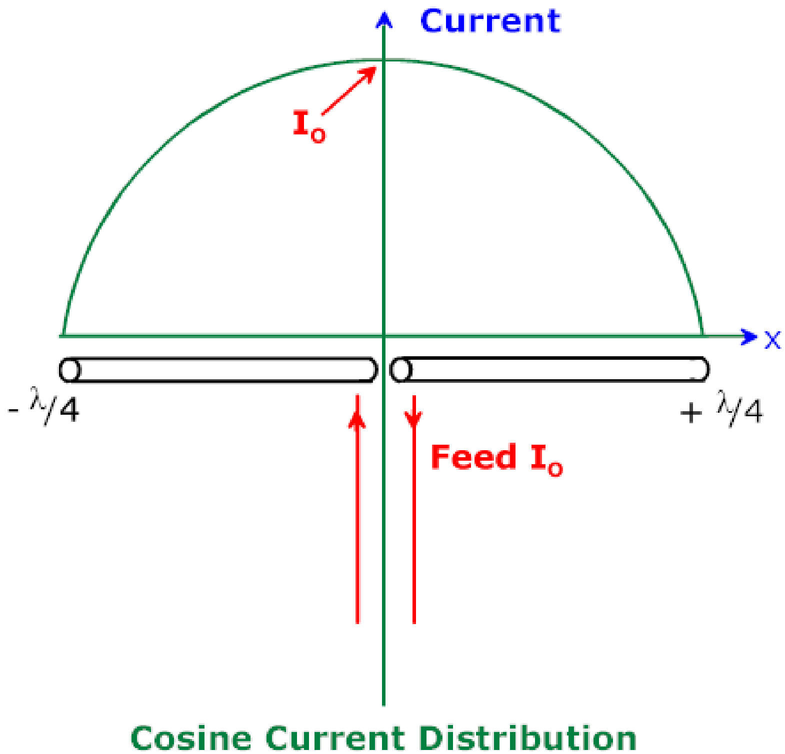

Figure 3
Sketch cosine current along dipole

In a half-wave antenna the current varies cosinusoidally along the length of the wire, with a maximum at the feed. See **Figure 3**. To find the "effective resistance" which the antenna loss presents to the feed, we need to multiply the resistance of a uniform-current length of wire by 0.707 or $1/\sqrt{2}$, because less current flows as we move further away from the centre, due to the radiation mechanism. If we take our formula for R above, and substitute $\lambda/2$ for the length L and incorporate the 0.707 factor, we obtain

$$R = \rho\lambda/(2\sqrt{2}\pi D\delta) \text{ ohms}$$

so then we can combine this with our earlier result for the skin depth to find

$$R = (1/D)\sqrt{15\rho\lambda\mu_r} \text{ ohms}$$

where we now notice that the diameter D of the wire or rod is the most significant factor which we might try to alter to reduce the resistive loss R, which rises gently with wavelength λ as the square root of the wavelength. We also see that the resistive loss R scales as the square root of the relative permeability μ_r, and as the square root of the electrical resistivity ρ.

Half wave dipole antenna	
Wavelength metres	Loss resistance ohms
160	3.21
80	2.27
40	1.61
20	1.13
10	0.80
2	0.36
Pure copper 2mm diameter wire (AWG12)	

AWG table		
Diameters		
AWG	inches	mm
OOOO	0.46	11.684
OOO	0.4096	10.40384
OO	0.3648	9.26592
0	0.3249	8.25246
1	0.2893	7.34822
2	0.2576	6.54304
3	0.2294	5.82676
4	0.2043	5.18922
5	0.1819	4.62026
6	0.162	4.1148
7	0.1443	3.66522
8	0.1285	3.2639
9	0.1144	2.90576
10	0.1019	2.58826
11	0.0907	2.30378
12	0.0808	2.05232
13	0.072	1.8288
14	0.0641	1.62814
15	0.0571	1.45034
16	0.0508	1.29032
17	0.0453	1.15062
18	0.0403	1.02362
19	0.0359	0.91186
20	0.032	0.8128
21	0.0285	0.7239
22	0.0254	0.64516
23	0.0226	0.57404

Note that the formulas in this article are all METRIC so the diameter D is in metres. For reference, 1 metre = 1000mm = 39.37 inches.

Now we need a table of electrical resistivities ρ of common metals. These numbers are in nano-ohms.metres and should be multiplied by 10^{-9} to be used in the formulas above

Silver	15.9
Copper	17.2
Brass	about 39, check with supplier
Tin	110
Lead	210
Gold	22
Aluminium	26.5
Aluminium alloy	about 30 to about 60, check with manufacturer
Iron	97.1
Stainless steel 304	72
Tin-lead solder	134 to about 200 depending on composition
Zinc	59

It is interesting how much adverse difference using tinned copper wire, coated with tin-lead solder, makes to the electrical resistance of half wave dipoles. The table below displays this.

Half wave dipole antenna	
Wavelength metres	Loss resistance ohms
160	9.29
80	6.57
40	4.66
20	3.27
10	2.31
2	1.04
Tinned copper 2mm diameter wire (|AWG 12)	

In the next table, we take a representative wavelength of 40 metres and list the loss resistances of half wave dipoles made from various materials. For the sake of this table we have assumed that the relative permeability of all metals except iron is 1.0, and that of iron is 1000, which will be quite variable depending on how it has been treated.

Half wave dipole antenna at 40 metres wavelength	
Material	Loss resistance R_{loss} (ohms)
Silver	1.55
copper	1.61
Brass	About 2.4
Tin	4.07
Lead	5.63
Gold	1.82
aluminium	2.00
Aluminium alloy	About 2.13 to 3.0
Iron	121
Stainless steel 304	3.29
Tin-lead solder	4.66
Zinc	2.98
2mm diameter wire (|AWG 12)	

At 160 metres wavelength, these figures for loss resistance should be doubled, and at 10 metres wavelength they should be halved.

Doubling the wire diameter to 6AWG halves these loss resistances. Halving the wire diameter to 18AWG doubles these loss resistances.

Discussion

This exercise has proved interesting to your author. As remarked earlier, because the loss resistances depend only on the square roots of the wavelength and of the resistivity of the antenna material, they are less sensitive to these parameters than one might have guessed. Magnetic permeability can be large and varies widely even between samples of the same magnetic material, depending on how the material has been treated, and over time as the material is exposed to the (electromagnetic and temperature) environment. Many stainless steels are magnetic, so it is important to check for this before choosing them as a potential antenna material.

To wrap up this short article, we present a table of maximum possible efficiencies for 12AWG wire half wave dipole antennas in free space made of these various materials. Figures are given for 160m, 40m, and 10m and it is assumed that the radiation resistance Rrad is 72 ohms for each half-wave antenna irrespective of frequency, and that the efficiency η is defined by the formula

$$\eta = 100(R_{rad})/(R_{loss} + R_{rad}) \text{ percent}$$

Maximum Efficiencies (percent) of half wave dipoles			
material	160m	40m	10m
Silver	95.9	97.9	98.9
Copper	95.7	97.8	98.9
Brass	about 93.8	about 96.8	about 98.4
Tin	89.8	94.7	97.3
Lead	86.5	92.8	96.7
Gold	95.2	97.5	98.8
Aluminium	94.7	97.3	98.6
Aluminium alloy	about 93.3	about 96.5	about 98.2
Iron	about 23	about 37	about 54
Stainless steel 304	91.6	95.6	97.8
Tin-lead solder	88.5	93.9	96.9
Zinc	92.4	96.0	98.0
12 AWG circular cross-section wire			

These efficiencies show that for non-magnetic materials at any HF wavelength of interest to Ham radio folk the transmit power lost in the resistance of a 12AWG wire antenna is at most 0.6 dB or so, and for copper at 160m is only 0.2 dB.

It is a good idea NOT to use magnetic materials, and possibly wires coated with tin, lead, or solder. Otherwise the material selection for wires 12 AWG or larger does not seem to be very important for ham radio people. In broadcast transmit antenna applications at longer wavelengths than 160 metres, it is usual to use multiple wires in a “cage dipole” configuration, with each wire appreciably fatter than 12 AWG.

Galvanised iron or steel (which is coated in zinc) is only about 3.4% less efficient than pure copper at the longest wavelength of 160 metres where the loss matters most, for this wire gauge.

Let’s look at this data another way. Suppose we feed half wave dipole antennas with 100W of RF power at these various wavelengths. Let us tabulate the number of watts of heat generated in the antennas for (bulk metal or coatings of) the various metals.

Watts of heat dissipated for 100W input power, half wave dipole			
material	160m	40m	10m
Silver	4.1	2.1	1.1
Copper	4.3	2.2	1.1
Brass	about 6.2	about 3.2	about 1.6
Tin	10.2	5.3	2.7
Lead	13.5	7.2	3.3
Gold	4.8	2.5	1.2
Aluminium	5.3	2.7	1.4
Aluminium alloy	about 6.7	about 3.5	about 1.8
Iron	about 77	about 63	about 46
Stainless steel 304	8.4	4.4	2.2
Tin-lead solder	11.5	6.1	3.1
Zinc	7.6	4.0	2.0
12 AWG circular cross-section wire			

Conclusions

I hope the readers will agree with me that this has been a useful exercise, even if it does just involve plugging numbers into a "well-known" formula. There is sufficient information here that readers can adapt the calculations to their own antenna applications; in particular, compact antennas having much lower tuned R_{rad} values come to mind. Even so, there does not seem to be much motivation to go beyond copper or aluminium for antenna materials; silver plating does not seem to be justified, especially as it has problems with tarnish, and gold plating is also not indicated.

To close, we repeat the admonition to avoid conducting materials with large relative permeability, which in practice means avoiding any magnetic material or material attracted by a magnet. In such materials the loss is large, and not quantifiable or stable over time. **–30-**

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